

Differential Equations

Example

1. A population model is given by $\frac{dP}{dt} = -10P(P-1)(P-5)^2$. How does the fate of the population depend on the initial population?

Problems

2. True False Given a differential equation $\frac{dP}{dt} = f(P)$, if we cannot solve for P , then we have no hope of describing P 's behavior.
3. True False If the rate of change of x is proportional to x , then $\frac{dx}{dt} = ax + b$.
4. True False The doubling time of the differential equation $\frac{dN}{dt} = 2t$ depends on the initial value of N .
5. Write a differential equation describing the fact that the rate of change of the mass M of an object is proportional to its cube root.
6. If $\frac{dN}{dt} = (-\ln 2)N$, then does the doubling time or halving time make more sense? Find it.
7. A population model is given by $\frac{dP}{dt} = (P-1)(P-10)$. How does the fate of the population depend on the initial population?

Separable Equations

Example

8. Find the solution to $\frac{dy}{dt} = 3t^2y^3 + e^ty^3$ with $y(0) = -1$.

Problems

9. True False We can always use the method of separable equations to solve $\frac{dy}{dt} = f(y, t)$.
10. True False The differential equation $\frac{dy}{dt} = y + t$ is separable.
11. Find the solution to $\frac{dy}{dt} = te^y$ with $y(0) = 1$.
12. Find the solution to $\frac{dy}{dx} = 6xy^2$ with $y(1) = 1/4$.
13. Find the solution to $\frac{dy}{dx} = \frac{3x^2+2x+1}{2y+1}$ with $y(0) = 1$.
14. Find the solution to $\frac{dr}{dt} = \frac{r^2}{t}$ with $r(1) = 1$.
15. Find the solution to $\frac{dy}{dt} = 2y + 3$ with $y(0) = 0$.
16. Find the solution to $\frac{dx}{dy} = e^{x-y}$ with $x(0) = 0$.